

Calibration of the interaction energy between Bose and Fermi superfluids

Ren Zhang,^{1,2} Wei Zhang,^{1,3} Hui Zhai,² and Peng Zhang^{1,3}

¹*Department of Physics, Renmin University of China, Beijing, 100872, China*

²*Institute for Advanced Study, Tsinghua University, Beijing, 100084, China*

³*Beijing Key Laboratory of Opto-electronic Functional Materials & Micro-nano Devices, 100872 (Renmin Univeristy of China)*

In this paper we study the interaction energy in a mixture of Bose and Fermi superfluids realized in recent cold atom experiment. On the Bose-Einstein-condensate (BEC) side of a Feshbach resonance between fermionic atoms, this interaction energy can be directly related to the scattering length between a bosonic atom and a dimer composed of fermions. We calculate the atom-dimer scattering length from a three-body analysis with both a zero-range model and a separable model including the van der Waals length scale, and we find significant deviation from the result given by a mean-field approach. We also find that the multiple scattering between atom and dimer can account for such a deviation. Our results provide a calibration to the mean-field interaction energy, which can be verified by measuring the shift of collective oscillation frequency.

I. INTRODUCTION

Few-body problems play many important roles in the study of cold atom gases, partly due to the diluteness condition which is generally fulfilled in the underlying systems. One of the most important examples is that few-body results, which can be numerically exact, can provide benchmark and calibration of many-body theories, where approximations are usually inevitable. For instance, in the study of BEC-BSC crossover of a Fermi superfluid around a Feshbach resonance, the mean-field theory yields that the scattering length between fermion pairs (or dimers) equals $2a_s$, with a_s the scattering length between fermionic particles [1, 2]. However, a precise four-body calculation gives a result of $0.6a_s$ [3]. This significant deviation suggests that it is necessary to include pair fluctuations in order to obtain a more accurate many-body description. Indeed, pair fluctuation theory can reduce this dimer scattering length to $0.75a_s$ [4], which is much closer to the few-body result. Moreover, experimental measurements of interaction energy have confirmed the prediction from few-body calculation [5].

Recently, the ENS Group has realized the first mixture of Bose and Fermi superfluids with bosonic ^7Li and two spin components of ^6Li [6]. To reach Fermi superfluid, the magnetic field has to be tuned around a Feshbach resonance between two fermionic components. In this regime all other interaction parameters are small. For instance, in this experiment the boson-fermion scattering length a_{bf} is only $40.8a_0$ (where a_0 is the Bohr radius). With such a weak interaction between bosons and fermions, one may expect that the interaction energy between Bose and Fermi superfluids can be obtained by mean-field theory quite accurately. In fact, such a mean-field and hydrodynamic theory treatment have been used in analyzing this system [6–9]. The purpose of this work is to provide a calibration of this mean-field theory from few-body calculation of atom-dimer scattering length.

Considering the situation that boson-fermion scattering length a_{bf} is independent of fermionic spin species (as

in the current experiment), boson-fermion interaction is described by

$$\hat{V} = \frac{2\pi\hbar^2 a_{\text{bf}}}{m_{\text{bf}}} \sum_{\sigma} \int d^3\mathbf{r} \hat{b}^{\dagger}(\mathbf{r}) \hat{b}(\mathbf{r}) \hat{c}_{\sigma}^{\dagger}(\mathbf{r}) \hat{c}_{\sigma}(\mathbf{r}), \quad (1)$$

where \hat{b}^{\dagger} and $\hat{c}_{\sigma}^{\dagger}$ are creation operators for bosons and fermions with spin $\sigma = \uparrow$ or \downarrow , respectively, and m_{bf} is the reduced mass of the boson and fermion. With Hartree-Fock mean-field decomposition, $\langle \hat{b}^{\dagger} \hat{b} \rangle = n_{\text{b}}$ and $\langle \hat{c}_{\sigma}^{\dagger} \hat{c}_{\sigma} \rangle = n_{\text{f}}$, this interaction energy density is naturally given by

$$\mathcal{E} = \frac{2\pi\hbar^2 a_{\text{bf}}}{m_{\text{bf}}} n_{\text{b}} n_{\text{f}}. \quad (2)$$

On the other hand, on the BEC side of the resonance, the Fermi superfluid can be viewed as a Bose condensate of dimers. The interaction energy between Bose and Fermi superfluids can then be considered as the boson-dimer interaction. We introduce a_{ad} as the scattering length between bosonic atoms and dimers. In the dilute limit, the interaction energy density is given by

$$\mathcal{E} = \frac{2\pi\hbar^2 a_{\text{ad}}}{m_{\text{ad}}} n_{\text{b}} n_{\text{d}}, \quad (3)$$

where n_{d} is density of dimers, $n_{\text{d}} = n_{\text{f}}/2$, and m_{ad} is the reduced mass of the bosonic atom and dimer.

Equations (2) and (3) are two different ways of representing the same interaction energy. Therefore, by equating these two expressions, we can obtain the atom-dimer scattering length from mean-field theory as

$$a_{\text{ad}}^0 = \frac{2m_{\text{ad}}}{m_{\text{bf}}} a_{\text{bf}}. \quad (4)$$

In this paper we will determine precisely the atom-dimer scattering length from a three-body calculation. Its deviation from Eq. (4) will be used as a calibration of the mean-field theory. Our calculation focuses on the resonance regime of Fermi superfluid with $a_{\text{bf}}/a_{\text{f}} \ll 1$ (where a_{f} denotes the scattering length between two

fermionic components). Besides, we also retain ourselves within the case of $a_{\text{bf}} > 0$ to ensure the stability of this system) [10]. To our surprise, it is found that even for weak boson-fermion interaction where $a_{\text{bf}}/a_{\text{f}}$ is only a few percent (for instance, for typical fermion density $n_{\text{f}} = 1.33 \times 10^{13} \text{cm}^{-3}$, $a_{\text{bf}} = 40.8a_0$, and when $1/(k_{\text{F}}a_{\text{f}}) = 1$, $a_{\text{bf}}/a_{\text{f}} = 0.01$), the difference between outcomes of three-body calculation and mean-field approach can be as large as 10% - 30%, depending on the mass ratio of boson to fermion. Our results give a correction to mean-field interaction energy on the BEC side of the fermionic Feshbach resonance, and can be verified by a frequency shift of collective dipole oscillation, as done in a recent experiment [6].

II. ATOM-DIMER SCATTERING LENGTH

A. STM-equation approach

The atom-dimer scattering length can be obtained by solving the Schrödinger equation for the system composed of a bosonic atom and two fermionic atoms with different spins. A similar method has been applied to a two-component Fermi system to obtain the fermion-dimer scattering length $1.2a_{\text{s}}$ [11, 12]. In our case, we label the two fermions as 1 and 2, and the boson as atom 3. The Hamiltonian for this three-body system is then given by $H = T + V_{12} + V_{23} + V_{31} \equiv T + V$, where T is the kinetic energy and V_{ij} is the interaction potential between atoms i and j . We model the interaction V_{ij} by

the Huang-Yang pseudopotential [13]

$$V_{ij} = \frac{2\pi\hbar^2 a_{ij}}{m_{ij}} \delta(\mathbf{r}_{ij}) \frac{\partial}{\partial r_{ij}}(r_{ij}), \quad (5)$$

with a_{ij} the scattering length between the atoms i and j (i.e., $a_{12} = a_{\text{f}}$ and $a_{31} = a_{23} = a_{\text{bf}}$), and \mathbf{r}_{ij} and m_{ij} the relative coordinate and reduced mass of these two atoms, respectively.

When $a_{\text{f}} > 0$, two fermionic atoms can form a dimer with binding energy $\hbar^2/(2m_{12}a_{\text{f}}^2)$ and bound-state wave function $|\phi_{\text{b}}\rangle_{12}$. The atom-dimer scattering length a_{ad} is defined as [14]

$$a_{\text{ad}} = 4\pi^2\hbar m_{\text{ad}} \langle \Psi_{\text{in}} | (V_{23} + V_{31}) | \Psi_{+} \rangle. \quad (6)$$

Here, $|\Psi_{\text{in}}\rangle$ is the incident state of the atom-dimer scattering process which can be expressed as $|\Psi_{\text{in}}\rangle = |\phi_{\text{b}}\rangle_{12}|0\rangle_{3-12}$, with $|0\rangle_{3-12}$ the eigenstate of the relative momentum between atom 3 and the center of mass of atoms 1 and 2, with eigenvalue zero. In Eq. (6), $|\Psi_{+}\rangle$ is the three-body scattering state related to $|\Psi_{\text{in}}\rangle$ via the Lippmann-Schwinger equation [14]:

$$|\Psi_{+}\rangle = \lim_{\varepsilon \rightarrow 0^{+}} \frac{i\varepsilon}{-\frac{\hbar^2}{2m_{12}a_{\text{f}}^2} + i\varepsilon - H} |\Psi_{\text{in}}\rangle. \quad (7)$$

With a straightforward calculation employing Eqs. (6) and (7) we obtain the Skorniakov-Ter-Martirosian (STM) equations [15] for our system (Appendix A). In the natural unit $\hbar = m_{\text{f}} = 1$ with m_{f} the mass of a fermionic atom, these equations are

$$\frac{\frac{M''}{M}a(K, \varepsilon)}{\frac{1}{a_{\text{f}}} + \sqrt{\frac{M''}{M}K^2 + \frac{1}{a_{\text{f}}^2} - i\varepsilon}} + \frac{2}{\pi^2} \int_0^{\Lambda e^{i\eta}} dK' \frac{K'}{K} \ln \left[\frac{\frac{M'}{2M}K^2 + KK' + \gamma_K}{\frac{M'}{2M}K^2 - KK' + \gamma_K} \right] \zeta(K', \varepsilon) = 0; \quad (8)$$

$$\left[\frac{1}{a_{\text{bf}}} - \frac{\sqrt{2MM'}}{M'} \sqrt{\frac{M''K^2}{2M'} + \frac{1}{a_{\text{f}}^2} - i\varepsilon} \right] \zeta(K, \varepsilon) - \frac{2M'}{M} \int_0^{\Lambda e^{i\eta}} dK' \frac{K'}{K[K'^2 - i(\frac{4M}{M''})\varepsilon]} \ln \left[\frac{\frac{M'K'^2}{2M} + KK' + \gamma_K}{\frac{M'K'^2}{2M} - KK' + \gamma_K} \right] a(K', \varepsilon) \\ + \frac{M'}{2\pi} \int_0^{\Lambda e^{i\eta}} dK' \frac{K'}{K} \ln \left[\frac{\frac{M'}{2M}(K^2 + K'^2) + \frac{1}{M}KK' + \frac{1}{a_{\text{f}}^2} - i\varepsilon}{\frac{M'}{2M}(K^2 + K'^2) - \frac{1}{M}KK' + \frac{1}{a_{\text{f}}^2} - i\varepsilon} \right] \zeta(K', \varepsilon) = \frac{2\pi M'}{M} \left[\frac{i\varepsilon}{\gamma_K(\gamma_K + i\varepsilon)} - \frac{1}{\gamma_K} \right], \quad (9)$$

where $M = m_{\text{b}}/m_{\text{f}}$ with m_{b} the mass of a bosonic atom, $M' = M + 1$, $M'' = M + 2$ and $\gamma_K = K^2 + a_{\text{f}}^{-2} - i\varepsilon$. As shown in Appendix A, after solving Eqs. (8) and (9) we can obtain the atom-dimer scattering length a_{ad} via the relation $a_{\text{ad}} = \lim_{\varepsilon \rightarrow 0^{+}} a(0, \varepsilon)$. The atom-dimer scattering length has also been studied in other systems [17–28].

We note that in Eqs. (8) and (9), a high-energy cutoff $\Lambda e^{i\eta}$ is required in order to regularize integrations. The

exact value of Λ and η is determined by the detail of van der Waals interaction potential between two atoms. It is now known that Λ is of the order of the van der Waals length [29–35], and η is usually very small. Since for weak boson-fermion interaction, a_{bf} is also of the order of the van der Waals length, we have varied Λ between $2/a_{\text{bf}}$ and $6/a_{\text{bf}}$ and varied η between 0 and 0.08.

It is found that the final result in the regime we are interested in is very insensitive to Λa_{bf} and η . We also

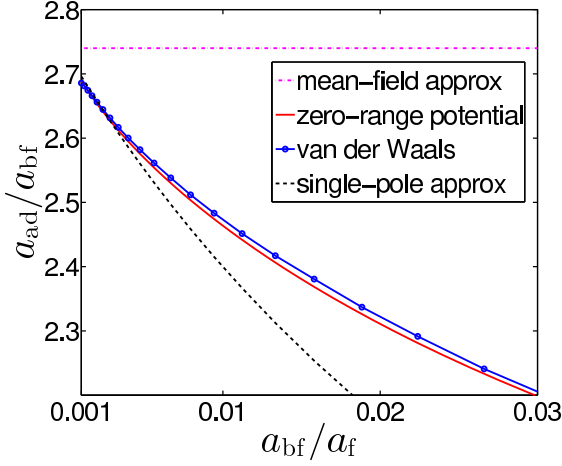


FIG. 1: (Color online) The atom-dimer scattering length a_{ad}/a_f as a function of a_{bf}/a_f given by three-body calculation with mean-field approximation (purple dash-dotted line), zero-range pseudo potential (red solid line), single-pole approximation (black dashed line), and three-body calculation with a separable potential suggested in Ref. [34] to incorporate the van der Waals effect (blue circle with solid line). Here we illustrate the results for ${}^6\text{Li}$ - ${}^7\text{Li}$ mixture with $a_{\text{bf}} = 40.8a_0$, $\Lambda = 4/a_{\text{bf}}$, $\eta = 0$, $R_{\text{vdW}} = 31.26a_0$ for ${}^6\text{Li}$ - ${}^6\text{Li}$ interaction, and $R_{\text{vdW}} = 32.49a_0$ for ${}^6\text{Li}$ - ${}^7\text{Li}$ interaction [36–38].

note that in the zero-range model, a boson can form a two-body bound state with one of the fermions with binding energy $1/(2m_{\text{bf}}a_{\text{bf}}^2)$. It is then possible for a dimer composed of two fermions to break up after scattering with a boson, and one fermion paired with this boson to form such a boson-fermion bound state. This inelastic atom-dimer scattering process will also give rise to an imaginary part for a_{ad} . However, within the regime we are interested in, the boson-fermion bound state is deeply bounded with very small overlap with the incident state. As a consequence, the inelastic scattering cross-section is extremely small and the imaginary part of a_{ad} remains negligible.

The results of a_{ad} for a ${}^6\text{Li}$ - ${}^7\text{Li}$ mixture is shown by the red solid line in Fig. 1. Here we fix a_{bf} and calculate $a_{\text{ad}}/a_{\text{bf}}$ with varying a_{bf}/a_f . Comparing with the mean-field result shown by the horizontal dash-dotted line, one can see that the two approaches agree with each other only when $a_{\text{bf}}/a_f \rightarrow 0$. For finite and positive a_{bf}/a_f , the three-body result is always below the mean-field expectation. Specifically, the deviation is already about 10% for $a_{\text{bf}}/a_f \approx 0.01$, and keeps increasing with a_{bf}/a_f until a_{bf} becomes comparable to a_f where Efimov physics starts to set in.

In Fig. 2 we investigate this deviation for other possible realizations of Bose-Fermi mixtures, including ${}^6\text{Li}$ - ${}^{87}\text{Rb}$, ${}^6\text{Li}$ - ${}^{133}\text{Cs}$, ${}^{40}\text{K}$ - ${}^{23}\text{Na}$, and ${}^{40}\text{K}$ - ${}^7\text{Li}$. Here, we plot the relative derivation of the three-body result from the mean-field value as $(a_{\text{ad}}^0 - a_{\text{ad}})/a_{\text{ad}}^0$. We find, on one hand, they all have qualitatively the same behaviors, and

on the other hand, the deviation increases with enhanced boson-fermion mass ratio.

In the following, we would like to further understand two questions: first, what is the major physical process that causes such a significant deviation; second, since a_{bf} is comparable to van der Waals length, whether a van der Waals potential will change this result obtained from zero-range models. The first question is to understand the underlying physics better, while the second question is crucial when comparing with experiments.

B. Born and single-pole approximations

To understand more about the difference between a_{ad} and a_{ad}^0 , we can rewrite Eq. (7) as

$$|\Psi_+\rangle = |\Psi_{\text{in}}\rangle + G_3(V_{23} + V_{31})|\Psi_+\rangle, \quad (10)$$

where $G_3 = [-\hbar^2/(2m_{12}a_f^2) + i0^+ - (T + V_{12})]$ is the Green's operator for a free boson and two interacting fermions, and expand Eq. (6) as

$$\begin{aligned} a_{\text{ad}} = & 4\pi^2\hbar m_{\text{ad}}\langle\Psi_{\text{in}}|(V_{23} + V_{31})|\Psi_{\text{in}}\rangle \\ & + 4\pi^2\hbar m_{\text{ad}}\langle\Psi_{\text{in}}|(V_{23} + V_{31})G_3(V_{23} + V_{31})|\Psi_{\text{in}}\rangle + \dots \end{aligned} \quad (11)$$

If we take the first-order Born approximation by neglecting all higher-order terms in Eq. (11), it is straightforward to show that

$$a_{\text{ad}} = 4\hbar\pi^2 m_{\text{ad}}\langle\Psi_{\text{in}}|(V_{23} + V_{31})|\Psi_{\text{in}}\rangle = \frac{2m_{\text{ad}}}{m_{\text{bf}}}a_{\text{bf}} = a_{\text{ad}}^0. \quad (12)$$

This means that a_{ad}^0 deduced from many-body mean-field treatment is equivalent to that obtained from a three-body calculation with first-order Born approximation. Thus, the difference between the exact a_{ad} and the mean-field a_{ad}^0 is due to processes beyond the first-order Born approximation.

The higher-order terms of Eq. (11) correspond to the following two types of atom-dimer scattering processes: (i) a dimer composed of two fermions remains in the bound state, and undergoes repeated collisions with bosons; (ii) an incoming dimer first breaks into two fermions in the scattering state, and then they return to a bound state after the second collision. While the processes of the second type are difficult to incorporate, the ones of the first type can be integrated using a “single-pole” approximation. Within this approach, the three-body Green's function G_3 in Eq. (11) is approximated as

$$G_3 \approx \int d\mathbf{K} \frac{|\mathbf{K}\rangle_{3-12}\langle\mathbf{K}| \otimes |\phi_b\rangle_{12}\langle\phi_b|}{i0^+ - \frac{K^2}{2m_{\text{ad}}}}, \quad (13)$$

where $|\mathbf{K}\rangle_{3-12}$ is the eigenstate of the relative momentum between atom 3 and the center of mass of atoms 1 and 2.

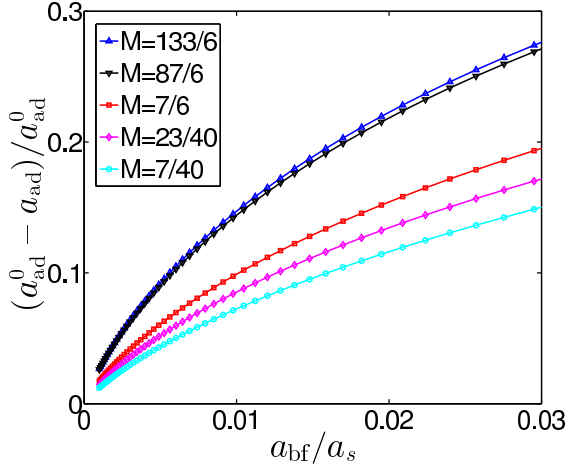


FIG. 2: (Color online) The relative deviation $(a_{\text{ad}}^0 - a_{\text{ad}})/a_{\text{ad}}^0$ for different mass ratios ($M \equiv m_b/m_f$). Cases shown are, from top to bottom, $^{133}\text{Cs}-^6\text{Li}$ (133/6, blue line with triangle), $^{87}\text{Rb}-^6\text{Li}$ (87/6, black line with inverted triangle), $^7\text{Li}-^6\text{Li}$ (7/6, red line with square), $^{23}\text{Na}-^{40}\text{K}$ (23/40, purple line with diamond), and $^7\text{Li}-^{40}\text{K}$ (7/40, cyan line with circle).

As a result, the wave function for relative motion between atoms 1 and 2 is forced in the bound state. With this approximation, process (i) is fully taken into account, while process (ii) is neglected.

As shown in Appendix B, under the single-pole approximation we can obtain the atom-dimer scattering length a_{ad} via the relation ($\hbar = m_f = 1$) $a_{\text{ad}} = \lim_{\varepsilon \rightarrow 0^+} \left(\frac{8M\pi^2}{M+2} \right) T(0, \varepsilon)$. Here the function $T(K, \varepsilon)$ satisfies the integral equation

$$\begin{aligned} T(K, \varepsilon) &+ \frac{8(M+1)a_{\text{bf}}}{(M+2)\pi^2 a_f} \int dK' \mathcal{F}(K, K', \varepsilon) T(K', \varepsilon) \\ &= \frac{(M+1)a_{\text{bf}}}{M\pi^3 a_f} \int_0^\infty \frac{k' dk'}{K \gamma_{k'}} \log \left[\frac{4 + a_f^2 (K + 2k')^2 - 4i\varepsilon}{4 + a_f^2 (K - 2k')^2 - 4i\varepsilon} \right], \end{aligned} \quad (14)$$

where the function $\mathcal{F}(K, K', \varepsilon)$ is defined as

$$\begin{aligned} \mathcal{F}(K, K', \varepsilon) &= \int_{-1}^1 du \int_0^\infty dk' \frac{k'}{\zeta(K, K', u) \gamma_{k'}} \times \\ &\quad \log \left[\frac{4 + a_f^2 [\zeta(K, K', u) + 2k']^2 - 4i\varepsilon}{4 + a_f^2 [\zeta(K, K', u) - 2k']^2 - 4i\varepsilon} \right], \end{aligned} \quad (15)$$

with $\zeta(K, K', u) = \sqrt{K^2 - 2uKK' + K'^2}$ and $\gamma_{k'} = k'^2 + a_f^{-2} - i\varepsilon$.

The result of a_{ad} from this single-pole approximation is shown as the black dashed line in Fig. 1 for the $^6\text{Li}-^7\text{Li}$ mixture, which has the same qualitative behavior as the exact three-body calculation and also gives a large deviation from the mean-field value a_{ad}^0 . This suggests that processes of type (i) are important processes which significantly reduce a_{ad} from a_{ad}^0 . This also shows that

in a more accurate many-body theory, it is necessary to include the ladder diagram to describe repeated scattering between fermion pairs and bosons. It is reminiscent of the ladder diagram between fermion pairs that reduces dimer-dimer scattering length significantly below its mean-field value on the BEC side of the Feshbach resonance [4].

C. Effect of van der Waals potential

To investigate the effect of van der Waals potential, we implement the separable potential proposed in Ref. [34]. In this method, the interaction potential between atoms i and j is modeled as

$$V_{ij} = \xi |\chi\rangle_{ij} \langle \chi|, \quad (16)$$

with $|\chi\rangle_{ij}$ a state for the relative motion of these two atoms. The potential V_{ij} is designed to reproduce not only the two-atom scattering length a_{ij} but also the s -wave zero-energy scattering wave function $u(r)/r$ in a van der Waals potential $-C_6/r^6$. To meet this requirement, the state $|\chi\rangle_{ij}$ and the parameter ξ should be chosen as $_{ij}\langle \mathbf{q} | \chi \rangle_{ij} = (2\pi)^{-3/2} \{1 - q \int_0^\infty dr [1 - \frac{r}{a_{ij}} - u(r)] \sin(qr)\}$ and $\xi = 4\pi [\frac{1}{a_{ij}} - \frac{2}{\pi} \int_0^\infty dq |_{ij}\langle \mathbf{q} | \chi \rangle_{ij}|^2]^{-1}$, respectively [34], where $|\mathbf{q}\rangle_{ij}$ is the eigen-state of the relative momentum with eigen-value \mathbf{q} . This separable potential naturally includes the length scale of the van der Waals length $R_{\text{vdW}} = 1/2(mC_6/\hbar^2)^{1/4}$ through wave function $u(\mathbf{r})$. Besides, the three-body calculation with this potential does not require extra three-body parameters. Indeed, Ref. [34] used this model to show the three-body parameter depends on the van der Waals length universally.

Here we use this separable potential to calculate the atom-dimer scattering length a_{ad} defined in Eq. (6). Following the same strategy as in Appendix A, we straightforwardly derive integral equations which are quite similar to STM equations, and obtain a_{ad} via numerically solving these equations. Here we have used the facts that $R_{\text{vdW}} = 31.26a_0$ for $^6\text{Li}-^6\text{Li}$ interaction, and $R_{\text{vdW}} = 32.49a_0$ for $^6\text{Li}-^7\text{Li}$ interaction [36–38]. The result of a_{ad} from this separable potential is also shown in Fig. 1. We find that in the regime of our interest, the correction is visible but not significant.

III. EXPERIMENTAL PREDICTIONS

With three-body calculation of a_{ad} , we can provide a correction to the mean-field interaction energy between Bose and Fermi superfluids on the BEC side of the fermionic Feshbach resonance. This interaction energy has been extracted from a measurement of collective mode frequency shift in the underlying system [6],

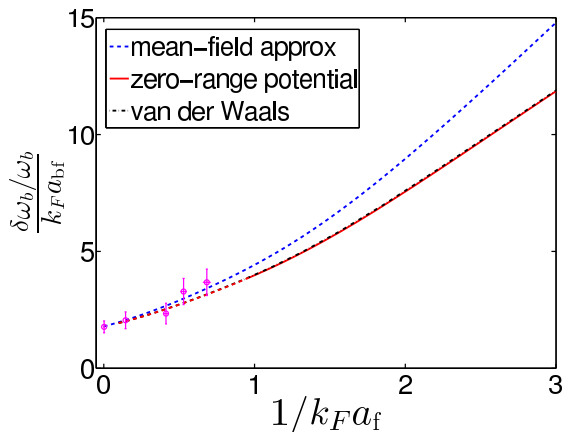


FIG. 3: (Color online) Frequency shift of dipole oscillation $\delta\omega_b$ for bosonic ^7Li atoms as a function of $1/(k_F a_f)$. Dots with error bar are experimental data [6]. Blue dashed line is predication from mean-field theory. Red solid line is predication based on a_{ad} calculated with zero-range model, and black dash-dotted line is predication based on a_{ad} from separable potential including the van der Waals effect. Here we have chosen $k_F = 4.6 \times 10^6 \text{m}^{-1}$. Other parameters take the same values as in Fig. 1.

where the condensate of bosonic atoms with smaller particle number is embedded in a larger cloud of Fermi superfluid. So the effective trapping potential experienced by bosons should include contributions from both the harmonic trap $V(\mathbf{r}) = (1/2)m\omega_b^2\mathbf{r}^2$ and the interaction energy between Bose and Fermi superfluids, leading to $V_{\text{eff}}(\mathbf{r}) = V(\mathbf{r}) + 2\pi\hbar^2 a_{\text{ad}} n_{\text{d}}(\mathbf{r})/m_{\text{ad}}$ on the BEC side of the Feshbach resonance. Here, since the number of fermions is much larger than that of bosons, we can safely assume that the fermion density distribution (i.e. dimer density distribution n_{d}) will not be affected by bosons and is solely determined by the equation of state of Fermi superfluid. Such an equation of state has been obtained quite accurately in previous experiment [39]. With the local density approximation, we have $n_{\text{d}}(\mathbf{r}) = n_{\text{d}}[\mu_{\text{f}} - V(\mathbf{r})]$, where we have used the fact that ^6Li and ^7Li experience almost identical trapping potentials. Thus, for the dipole oscillation of bosons, its frequency $\tilde{\omega}_b$ will be shifted away from ω_b , and the shift $\delta\omega_b = \omega_b - \tilde{\omega}_b$ is given by [6]

$$\frac{\delta\omega_b}{\omega_b} = \frac{\pi\hbar^2 a_{\text{ad}}}{m_{\text{ad}}} \left(\frac{dn_{\text{d}}}{d\mu_{\text{f}}} \right)_{\mathbf{r}=0}. \quad (17)$$

In Ref. [6], this collective frequency has been measured in the unitary regime (with $1/(k_F a_f) < 1$) and fitted with a mean-field theory by replacing a_{ad} with a_{ad}^0 in Eq. (17). The experimental data points and the mean-field fitting are shown in Fig. 3. However, in this regime, the size of dimers is even larger than the interparticle distance and the system can not be viewed as a boson-dimer mixture. Our expression of Eq. (17) should be applied to the regime with $1/(k_F a_f) > 1$, where the results for $\delta\omega_b/\omega_b$ with both zero-range and separable potential are

shown in Fig. 3 and compared with mean-field predication. Future experiments can perform more accurate measurements of frequency shift in the BEC regime; the difference between our result and the mean-field result can be distinguished.

IV. SUMMARY

In summary, our work studies the interaction energy between Bose and Fermi superfluids on the BEC side of the fermionic Feshbach resonance. Our result provides a striking example of how mean-field theory can be qualitatively inaccurate even for quite weak interaction strength, and suggests a route to improve many-body theory in this system. This result can be easily verified in the current experimental setup.

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Appendix A: STM Equations

In this Appendix we show our STM-equation approach for the calculation of atom-dimer scattering length a_{ad} . We will first introduce a function $a(\mathbf{K}, \varepsilon)$, and then prove that such a function satisfies the relation

$$\lim_{\varepsilon \rightarrow 0^+} a(0, \varepsilon) = a_{\text{ad}}. \quad (\text{A1})$$

Namely, one can obtain a_{ad} directly from $a(\mathbf{K}, \varepsilon)$. Then we will derive the equation for $a(\mathbf{K}, \varepsilon)$, i.e., Eqs. (8) and (9) in our main text. They are the STM equations of our system.

1. Function $a(\mathbf{K}, \varepsilon)$

As shown in the main text, our system includes two fermionic atoms (labeled 1 and 2) with the same mass and different spins, and one bosonic atom (labeled 3). In the following we use $|\cdot\rangle$ to denote the total quantum state of the relative motion of these three atoms, $|\cdot\rangle_{ij}$ for the state of the relative motion between atoms i and j , and $|\cdot\rangle_{i-jk}$ for the state of the relative motion between the atom i and the center of mass of atoms j and k . We further denote the mass of the fermionic atom and bosonic atom

as m_f and m_b , respectively. In our calculation we use the natural unit $\hbar = m_f = 1$. We model the two-body interaction with Huang-Yang pseudo potential, i.e., the interaction operator V_{ij} for the atoms i and j satisfies

$${}_{ij}\langle \mathbf{r} | V_{ij} | \Psi \rangle = \frac{2\pi a_{ij}}{m_{ij}} \delta(\mathbf{r}) \left[\frac{\partial}{\partial |\mathbf{r}|} (|\mathbf{r}| \times {}_{ij}\langle \mathbf{r} | \Psi \rangle) \right], \quad (\text{A2})$$

where $|\mathbf{r}\rangle_{ij}$ is the eigenstate of the relative position between atoms i and j with eigenvalue \mathbf{r} , and m_{ij} and a_{ij} are the reduced mass and scattering length of these two atoms, respectively. As shown in the main text, here we assume $a_{12} = a_f$ and $a_{31} = a_{23} = a_{bf}$.

To introduce the function $a(\mathbf{K}, \varepsilon)$, we first define state $|\psi_3(\varepsilon)\rangle_{3-12}$ via the relation

$${}_{12}\langle \mathbf{r} | V_{12} | \Psi_+(\varepsilon) \rangle = \delta(\mathbf{r}) |\psi_3(\varepsilon)\rangle_{3-12}, \quad (\text{A3})$$

where $|\Psi_+(\varepsilon)\rangle$ is defined as

$$|\Psi_+(\varepsilon)\rangle = \frac{i\varepsilon}{-a_f^{-2} + i\varepsilon - (T + V_{12} + V_{23} + V_{31})} |\Psi_{\text{in}}\rangle \quad (\text{A4})$$

with T the kinetic energy of the relative motion of these three atoms and $|\Psi_{\text{in}}\rangle$ the incident state of the atom-dimer scattering process, as defined in the main text. We further define function $\eta(\mathbf{K}, \varepsilon)$ as

$$\eta(\mathbf{K}, \varepsilon) = {}_{3-12}\langle \mathbf{K} | \psi_3(\varepsilon) \rangle_{3-12}. \quad (\text{A5})$$

With the aid of Eqs. (A3), (A4), and (A5), we can define the function $a(\mathbf{K}, \varepsilon)$ via the equation

$$\eta(\mathbf{K}, \varepsilon) = -\frac{\sqrt{2}}{\pi^{3/2} a_f^{1/2}} \left[2\pi^2 \delta(\mathbf{K}) + \frac{a(\mathbf{K}, \varepsilon)}{\left(\frac{4M}{M+2}\right) i\varepsilon - K^2} \right], \quad (\text{A6})$$

where $M = m_b/m_f$ and $|\mathbf{K}\rangle_{k-ij}$ is the eigenstate of the relative momentum between atom k and the center of mass of atoms i and j , with eigenvalue \mathbf{K} .

2. Relation Between $a(\mathbf{K}, \varepsilon)$ and a_{ad}

Now we prove the relation of Eq. (A1) which links the function $a(\mathbf{K}, \varepsilon)$ defined in Eq. (A6) with the atom-dimer scattering length a_{ad} . To this end we rewrite Eq. (A4) as

$$|\Psi_+(\varepsilon)\rangle = |\Psi_{\text{in}}\rangle + G_3(\varepsilon)(V_{23} + V_{31})|\Psi_+(\varepsilon)\rangle, \quad (\text{A7})$$

where $G_3(\varepsilon) = [-a_f^{-2} + i\varepsilon - (T + V_{12})]^{-1}$ is the Green's operator for the free bosonic atom together with the two interacting fermionic atoms taking the form

$$G_3(\varepsilon) = \int d\mathbf{K} \frac{|\mathbf{K}\rangle_{3-12} \langle \mathbf{K}| \otimes |\phi_b\rangle_{12} \langle \phi_b|}{i\varepsilon - \left(\frac{M+2}{4M}\right) K^2} + \int d\mathbf{K} d\mathbf{k} \frac{|\mathbf{K}\rangle_{3-12} \langle \mathbf{K}| \otimes |\mathbf{k}+\rangle_{12} \langle \mathbf{k}+|}{-a_f^{-2} + i\varepsilon - \left(\frac{M+2}{4M}\right) K^2 - \mathbf{k}^2}. \quad (\text{A8})$$

Here $|\mathbf{k}+\rangle_{12}$ is the scattering state of the relative motion between atoms 1 and 2 with incident momentum \mathbf{k} , and $|\phi_b\rangle_{12}$ is the bound state of these two atoms, as defined in our main text. By writing down Eq. (A8), we have used the fact that $T = \mathbf{p}_{12}^2 + \left(\frac{M+2}{4M}\right) \mathbf{P}_{3-12}^2$, where \mathbf{p}_{12} is the relative momentum operator between atoms 1 and 2, and \mathbf{P}_{3-12} is the relative momentum operator of atom 3 and the center of mass of atoms 1 and 2. We have also used the eigen equations

$$(\mathbf{p}_{12}^2 + V_{12}) |\phi_b\rangle_{12} = -\frac{1}{a_f^2} |\phi_b\rangle_{12}, \quad (\text{A9})$$

$$(\mathbf{p}_{12}^2 + V_{12}) |\mathbf{k}+\rangle_{12} = \mathbf{k}^2 |\mathbf{k}+\rangle_{12} \quad (\text{A10})$$

satisfied by $|\mathbf{k}+\rangle_{12}$ and $|\phi_b\rangle_{12}$.

Now we are at the stage to calculate $\eta(\mathbf{K}, \varepsilon)$. According to Eqs. (A2) and (A3), we have

$$\eta(\mathbf{K}, \varepsilon) = 4\pi a_f \left\{ \frac{\partial}{\partial |\mathbf{r}|} [|\mathbf{r}| \times {}_{12}\langle \mathbf{r} | ({}_{3-12}\langle \mathbf{K} | \Psi_+(\varepsilon) \rangle)] \right\}_{\mathbf{r}=0}. \quad (\text{A11})$$

Substituting Eqs. (A7) and (A8) into Eq. (A11), and using the results in two-body problems

$${}_{12}\langle \mathbf{r} | \phi_b \rangle_{12} = \frac{1}{\sqrt{2} a_f \pi} \frac{e^{-|\mathbf{r}|/a_f}}{|\mathbf{r}|}, \quad (\text{A12})$$

$${}_{12}\langle \mathbf{r} | \mathbf{k}+ \rangle_{12} = \frac{1}{(2\pi)^{3/2}} \left[e^{i\mathbf{k}\cdot\mathbf{r}} + \left(\frac{-1}{i|\mathbf{k}| + \frac{1}{a_f}} \right) \frac{e^{i|\mathbf{k}||\mathbf{r}|}}{|\mathbf{r}|} \right], \quad (\text{A13})$$

as well as the fact that $|\Psi_{\text{in}}\rangle = |\phi_b\rangle_{12} |0\rangle_{3-12}$, with $|0\rangle_{3-12}$ the eigenstate of \mathbf{P}_{3-12} with eigenvalue zero, we obtain

$$\begin{aligned} \eta(\mathbf{K}, \varepsilon) = & -\frac{2\sqrt{2}\pi}{\sqrt{a_f}} \delta(\mathbf{K}) - \frac{2\sqrt{2}\pi}{\sqrt{a_f}} \frac{{}_{12}\langle \phi_b | {}_{3-12}\langle \mathbf{K} | (V_{23} + V_{31}) | \Psi_+(\varepsilon) \rangle}{i\varepsilon - \left(\frac{M+2}{4M}\right) K^2} \\ & - \int d\mathbf{k} \frac{ia_f \sqrt{2} \times {}_{12}\langle \mathbf{k}+ | {}_{3-12}\langle \mathbf{K} | (V_{23} + V_{31}) | \Psi_+(\varepsilon) \rangle}{\sqrt{\pi} (-i + a_f |\mathbf{k}|) \left[-\frac{1}{a_f^2} + i\varepsilon - \left(\frac{M+2}{4M}\right) K^2 - \mathbf{k}^2 \right]}. \end{aligned} \quad (\text{A14})$$

Comparing Eq. (A14) with the definition of $a(\mathbf{K}, \varepsilon)$ Eq. (A6), we find that $a(\mathbf{K}, \varepsilon)$ can be re-expressed as

$$\begin{aligned} a(\mathbf{K}, \varepsilon) = & \left(\frac{8M\pi^2}{M+2} \right) [{}_{12}\langle \phi_b | {}_{3-12}\langle \mathbf{K} | (V_{23} + V_{31}) | \Psi_+(\varepsilon) \rangle] \\ & + \left[\left(\frac{4M}{M+2} \right) i\varepsilon - K^2 \right] g(\mathbf{K}, \varepsilon) \end{aligned} \quad (\text{A15})$$

where

$$\begin{aligned} g(\mathbf{K}, \varepsilon) = & \int d\mathbf{k} \frac{ia_f^{3/2} \pi \times {}_{12}\langle \mathbf{k}+ | {}_{3-12}\langle \mathbf{K} | (V_{23} + V_{31}) | \Psi_+(\varepsilon) \rangle}{(-i + a_f |\mathbf{k}|) \left[a_f^{-2} + i\varepsilon - \left(\frac{M+2}{4M}\right) K^2 - \mathbf{k}^2 \right]}. \end{aligned} \quad (\text{A16})$$

When $\mathbf{K} = 0$, in the limit $\varepsilon \rightarrow 0^+$ the second term in the right-hand side of Eq. (A15) becomes zero. Therefore, we have

$$\begin{aligned} & \lim_{\varepsilon \rightarrow 0^+} a(0, \varepsilon) \\ &= \left(\frac{8M\pi^2}{M+2} \right) \lim_{\varepsilon \rightarrow 0^+} [{}_{12}\langle \phi_b | {}_{3-12}\langle 0 | (V_{23} + V_{31}) | \Psi_+(\varepsilon) \rangle] \end{aligned} \quad (\text{A17})$$

On the other hand, according to Eqs. (5) and (6) in our main text, the atom-dimer scattering length a_{ad} is given by

$$a_{\text{ad}} = \lim_{\varepsilon \rightarrow 0^+} \left(\frac{8M\pi^2}{M+2} \right) {}_{12}\langle \phi_b | {}_{3-12}\langle 0 | (V_{23} + V_{31}) | \Psi_+(\varepsilon) \rangle. \quad (\text{A18})$$

Here we have used the fact that in our natural unit with $m_f = 1$, the atom-dimer reduced mass is $2M/(M+2)$. With Eq. (A18), we can rewrite Eq. (A17) as $\lim_{\varepsilon \rightarrow 0^+} a(0, \varepsilon) = a_{\text{ad}}$. That is the relation in Eq. (A1).

3. STM Equations

In the previous discussions, we show that the atom-dimer scattering length a_{ad} can be obtained directly from the function $a(\mathbf{K}, \varepsilon)$. Now we derive the equation for the function $a(\mathbf{K}, \varepsilon)$, i.e., the STM equation. To this end, we first define states $|\psi_1(\varepsilon)\rangle_{1-23}$ and $|\psi_2(\varepsilon)\rangle_{2-31}$ via relations

$${}_{23}\langle \mathbf{r} | V_{23} | \Psi_+(\varepsilon) \rangle = \delta(\mathbf{r}) |\psi_1(\varepsilon)\rangle_{1-23}, \quad (\text{A19})$$

$${}_{31}\langle \mathbf{r} | V_{31} | \Psi_+(\varepsilon) \rangle = \delta(\mathbf{r}) |\psi_2(\varepsilon)\rangle_{2-31}. \quad (\text{A20})$$

In our system, since atoms 1 and 2 have the same mass and scattering length with atom 3, it is easy to prove that ${}_{1-23}\langle \mathbf{K} | \psi_1(\varepsilon) \rangle_{1-23} = {}_{2-31}\langle \mathbf{K} | \psi_2(\varepsilon) \rangle_{2-31}$. We further define the function $\zeta(\mathbf{K}, \varepsilon)$ as

$$\begin{aligned} \zeta(\mathbf{K}, \varepsilon) &= - \left(2^{\frac{3}{2}} \pi^{\frac{5}{2}} \sqrt{a_f} \right) {}_{1-23}\langle \mathbf{K} | \psi_1(\varepsilon) \rangle_{1-23} \\ &= - \left(2^{\frac{3}{2}} \pi^{\frac{5}{2}} \sqrt{a_f} \right) {}_{2-31}\langle \mathbf{K} | \psi_2(\varepsilon) \rangle_{2-31}. \end{aligned} \quad (\text{A21})$$

As we have outlined in the previous section, it is easy to prove that

$$\begin{aligned} & \zeta(\mathbf{K}, \varepsilon) \\ &= -(2\pi)^{\frac{7}{2}} a_f^{\frac{1}{2}} a_{\text{bf}} \left\{ \frac{\partial}{\partial |\mathbf{r}|} [|\mathbf{r}| \times {}_{23}\langle \mathbf{r} | ({}_{1-23}\langle \mathbf{K} | \Psi_+(\varepsilon) \rangle)] \right\}_{|\mathbf{r}=0} \end{aligned} \quad (\text{A22})$$

$$= -(2\pi)^{\frac{7}{2}} a_f^{\frac{1}{2}} a_{\text{bf}} \left\{ \frac{\partial}{\partial |\mathbf{r}|} [|\mathbf{r}| \times {}_{31}\langle \mathbf{r} | ({}_{2-31}\langle \mathbf{K} | \Psi_+(\varepsilon) \rangle)] \right\}_{|\mathbf{r}=0} \quad (\text{A23})$$

by using Eqs. (A2), (A19), (A20) and (A21).

Now we rewrite Eq. (A4) as

$$|\Psi_+(\varepsilon)\rangle = i\varepsilon G_0(\varepsilon) |\Psi_{\text{in}}\rangle + G_0(\varepsilon) (V_{12} + V_{23} + V_{31}) |\Psi_+(\varepsilon)\rangle, \quad (\text{A24})$$

where

$$\begin{aligned} G_0(\varepsilon) &= \frac{1}{-a_f^{-2} + i\varepsilon - T} \\ &= \int d\mathbf{k} d\mathbf{K} \frac{|\mathbf{k}\rangle_{ij} \langle \mathbf{k}| \otimes |\mathbf{K}\rangle_{k-ij} \langle \mathbf{K}|}{-a_f^{-2} + i\varepsilon - [|\mathbf{k}|^2 + (\frac{M+2}{4M}) K^2]}, \end{aligned} \quad (\text{A25})$$

with $|\mathbf{k}\rangle_{ij}$ the eigenstate of \mathbf{p}_{ij} with eigenvalue \mathbf{k} and $K = |\mathbf{K}|$. Here (i, j, k) can take values $(1, 2, 3)$, $(2, 3, 1)$, or $(3, 1, 2)$. Substituting Eq. (A24) into Eq. (A11), and using Eqs. (A21), (A22) and (A23), we obtain

$$\begin{aligned} & \left(\frac{1}{a_f} - \sqrt{\left(\frac{M+2}{4M} \right) K^2 + \frac{1}{a_f^2} - i\varepsilon} \right) \eta(\mathbf{K}, \varepsilon) \\ & - \int \frac{d\mathbf{K}'}{2^{\frac{3}{2}} \pi^{\frac{9}{2}} \sqrt{a_f}} \frac{\zeta(\mathbf{K}', \varepsilon)}{\left[\left(\frac{M+1}{2M} \right) K^2 + K'^2 + \mathbf{K} \cdot \mathbf{K}' + \frac{1}{a_f^2} - i\varepsilon \right]} \\ & + \frac{2\sqrt{2}\pi}{\sqrt{a_f}} \left(\frac{1}{a_f} - \sqrt{\frac{1}{a_f^2} - i\varepsilon} \right) \delta(\mathbf{K}) = 0. \end{aligned} \quad (\text{A26})$$

Here we have used the relations

$$\begin{aligned} & {}_{12}\langle \mathbf{r} | ({}_{3-12}\langle \mathbf{K} | G_0(\varepsilon) V_{12} | \Psi_+(\varepsilon) \rangle) \\ &= \frac{\eta(\mathbf{K}, \varepsilon)}{(2\pi)^3} \int d\mathbf{k} \frac{e^{i\mathbf{k} \cdot \mathbf{r}}}{-\frac{1}{a_f^2} + i\varepsilon - [|\mathbf{k}|^2 + (\frac{M+2}{4M}) K^2]} \\ &= -\frac{\eta(\mathbf{K}, \varepsilon)}{(2\pi)^3} \int d\mathbf{k} \frac{e^{i\mathbf{k} \cdot \mathbf{r}}}{|\mathbf{k}|^2} + \frac{\eta(\mathbf{K}, \varepsilon)}{(2\pi)^3} \int d\mathbf{k} \left\{ \frac{e^{i\mathbf{k} \cdot \mathbf{r}}}{\Delta} + \frac{e^{i\mathbf{k} \cdot \mathbf{r}}}{|\mathbf{k}|^2} \right\} \\ &= -\frac{\eta(\mathbf{K}, \varepsilon)}{32\pi^4 |\mathbf{r}|} + \frac{\eta(\mathbf{K}, \varepsilon)}{(2\pi)^3} \int d\mathbf{k} \left\{ \frac{1}{\Delta} + \frac{1}{|\mathbf{k}|^2} \right\} + \mathcal{O}(|\mathbf{r}|), \end{aligned} \quad (\text{A27})$$

with $\Delta = -\frac{1}{a_f^2} + i\varepsilon - [|\mathbf{k}|^2 + (\frac{M+2}{4M}) K^2]$ and

$${}_{12}\langle \mathbf{k} | \phi_b \rangle_{12} = \frac{1}{\pi \sqrt{a_f}} \frac{1}{\left(|\mathbf{k}|^2 + \frac{1}{a_f^2} \right)}. \quad (\text{A28})$$

Substituting Eq. (A6) into Eq. (A26), we obtain

$$\begin{aligned} & \int \frac{d\mathbf{K}'}{(2\pi)^3} \frac{\zeta(\mathbf{K}', \varepsilon)}{\left[\left(\frac{M+1}{2M} \right) K^2 + K'^2 + \mathbf{K} \cdot \mathbf{K}' + \frac{1}{a_f^2} - i\varepsilon \right]} \\ & + \frac{\left(\frac{M+2}{8M} \right) a(\mathbf{K}, \varepsilon)}{\frac{1}{a_f} + \sqrt{\left(\frac{M+2}{4M} \right) K^2 + \frac{1}{a_f^2} - i\varepsilon}} = 0. \end{aligned} \quad (\text{A29})$$

On the other hand, by substituting Eq. (A24) into Eq. (A22) and using similar techniques as above, we obtain

$$\begin{aligned}
& \left[\frac{1}{a_{\text{bf}}} - \frac{\sqrt{2M(M+1)}}{M+1} \sqrt{\frac{M+2}{2(M+1)} K^2 + \frac{1}{a_f^2} - i\varepsilon} \right] \zeta(\mathbf{K}, \varepsilon) + \frac{(M+1)}{(2\pi)^2 M} \int d\mathbf{K}' \frac{\zeta(\mathbf{K}', \varepsilon)}{\frac{M+1}{2M}(K'^2 + K^2) + \frac{1}{M} \mathbf{K} \cdot \mathbf{K}' + \frac{1}{a_f^2} - i\varepsilon} \\
& - \frac{2\pi(M+1)}{M} \int \frac{d\mathbf{K}'}{(2\pi)^3} \frac{4\pi a(\mathbf{K}', \varepsilon)}{(K'^2 - \frac{4M}{M+2} i\varepsilon) \left(\frac{M+1}{2M} K'^2 + K^2 + \mathbf{K} \cdot \mathbf{K}' + \frac{1}{a_f^2} - i\varepsilon \right)} + \frac{2\pi(M+1)}{M(K^2 + \frac{1}{a_f^2} - i\varepsilon)} \\
& - \frac{2\pi(M+1)i\varepsilon}{M \left(K^2 + \frac{1}{a_f^2} - i\varepsilon \right) (K^2 + \frac{1}{a_f^2})} = 0.
\end{aligned} \tag{A30}$$

Equations (A29) and (A30) are the integral equations satisfied by $a(\mathbf{K}, \varepsilon)$ and $\zeta(\mathbf{K}, \varepsilon)$. We can further express $a(\mathbf{K}, \varepsilon)$ and $\zeta(\mathbf{K}, \varepsilon)$ as $a(\mathbf{K}, \varepsilon) = \sum_{l,m} a_{l,m}(K, \varepsilon) Y_l^m(\hat{\mathbf{K}})$ and $\zeta(\mathbf{K}, \varepsilon) = \sum_{l,m} \zeta_{l,m}(K, \varepsilon) Y_l^m(\hat{\mathbf{K}})$, where $\hat{\mathbf{K}}$ is the unit vector along the direction of \mathbf{K} , and Y_l^m is the spherical harmonic with degree l and order m . Then we can obtain equations of $a_{l,m}(K, \varepsilon)$ and $\zeta_{l,m}(K, \varepsilon)$. It is easy to prove that the equations for different values of (l, m) are decoupled with each other. Furthermore, the inhomogeneous term appears only in equations for $a_{0,0}(K, \varepsilon)$ and $\zeta_{0,0}(K, \varepsilon)$. As a result, we have $a_{l,m}(K, \varepsilon) = \zeta_{l,m}(K, \varepsilon) = 0$ for $l > 0$. Therefore, the solution of Eqs. (A29) and (A30) is independent of the direction of \mathbf{K} , and thus can be expressed as

$$a(\mathbf{K}, \varepsilon) = a(K, \varepsilon), \quad \zeta(\mathbf{K}, \varepsilon) = \zeta(K, \varepsilon). \tag{A31}$$

Using this result, Eqs. (A29) and (A30) can be simplified as Eqs. (8) and (9) in our main text. These two equations are the STM equations in our system. As shown in the main text, in our problem the three-body boundary condition in the region where all the three atoms are close with each other is necessary. Such a condition is provided by the momentum cutoff $\Lambda e^{i\eta}$ in the integrations in Eqs. (8) and (9) [40].

Appendix B: Single-Pole Approximation

In this Appendix we show our calculation of atom-dimer scattering length with single-pole approximation, and derive Eq. (14) in our main text. As we have discussed before, the atom-dimer scattering length is given by Eq. (A18):

$$a_{\text{ad}} = \lim_{\varepsilon \rightarrow 0^+} \left(\frac{8M\pi^2}{M+2} \right) {}_{12} \langle \phi_b | {}_{3-12} \langle 0 | (V_{23} + V_{31}) | \Psi_+(\varepsilon) \rangle, \tag{B1}$$

with $|\Psi_+(\varepsilon)\rangle$ given by Eq. (A7):

$$|\Psi_+(\varepsilon)\rangle = |\Psi_{\text{in}}\rangle + G_3(\varepsilon)(V_{23} + V_{31})|\Psi_+(\varepsilon)\rangle. \tag{B2}$$

Here the Green's function $G_3(\varepsilon)$ is defined in Eq. (A8), and the states and operators are defined as before.

As shown in main text, in the single-pole approximation, we have

$$G_3(\varepsilon) \approx \int d\mathbf{K} \frac{|\mathbf{K}\rangle {}_{3-12} \langle \mathbf{K} | \otimes |\phi_b\rangle {}_{12} \langle \phi_b |}{i\varepsilon - \left(\frac{M+2}{4M} \right) \mathbf{K}^2}. \tag{B3}$$

Substituting Eq. (B3) into Eq. (B2), we find that under the single-pole approximation there is

$$|\Psi_+(\varepsilon)\rangle = |\phi_b\rangle {}_{12} \langle \psi | {}_{3-12},$$

where $|\psi\rangle {}_{3-12}$ satisfies the relation

$${}_{3-12} \langle \mathbf{K} | \psi \rangle {}_{3-12} = \delta(\mathbf{K}) + \frac{T(\mathbf{K}, \varepsilon)}{i\varepsilon - \left(\frac{M+2}{4M} \right) \mathbf{K}^2} \tag{B4}$$

with

$$T(\mathbf{K}, \varepsilon) = {}_{3-12} \langle \mathbf{K} | {}_{12} \langle \phi_b | (V_{23} + V_{31}) | \Psi_+(\varepsilon) \rangle \tag{B5}$$

$$\begin{aligned}
& = \int d\mathbf{K}' [{}_{3-12} \langle \mathbf{K} | {}_{12} \langle \phi_b | (V_{23} + V_{31}) | \phi_b \rangle {}_{12} | \mathbf{K}' \rangle {}_{3-12}] \\
& \quad \times ({}_{3-12} \langle \mathbf{K}' | \psi \rangle {}_{3-12}).
\end{aligned} \tag{B6}$$

According to Eqs. (B1) and (B5), we have

$$a_{\text{ad}} = \lim_{\varepsilon \rightarrow 0^+} \left(\frac{8M\pi^2}{M+2} \right) T(0, \varepsilon). \tag{B7}$$

On the other hand, by substituting Eq. (B6) into Eq. (B4), we can obtain the integral equation for ${}_{3-12} \langle \mathbf{K} | \psi \rangle {}_{3-12}$, from which an integral equation for $T(\mathbf{K}, \varepsilon)$ can be derived. Using a similar analysis as in the paragraph before Eq. (A31), we can find that the solution of such an equation is independent of the direction of \mathbf{K} , i.e., we have $T(\mathbf{K}, \varepsilon) = T(K, \varepsilon)$, and the integral equation can be simplified as in Eq. (14) of our main text.

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